

Varianta 67

III.

13. a) $[12; 15; 18] = 2^2 \cdot 3^2 \cdot 5 = 180$.

$$b) \begin{cases} a = 12 \cdot c_1 + 6 \\ a = 15 \cdot c_2 + 9 \\ a = 18 \cdot c_3 + 12 \end{cases} \Rightarrow \begin{cases} a + 6 = 12(c_1 + 1) \\ a + 6 = 15(c_2 + 1) \\ a + 6 = 18(c_3 + 1) \end{cases} \Rightarrow a + 6 = [12; 15; 18] = 180 \Rightarrow a_{\min} = 174$$

14. a) $2x^2 + 3x + 1 = 0 \Rightarrow x_1 = -1; x_2 = -\frac{1}{2}$.

b) Dacă $x = 3 \Rightarrow 9m + (2m - 1) \cdot 3 + m - 1 = 0 \Rightarrow 16m = 4 \Rightarrow m = \frac{1}{4}$.

c) Dacă $m \in \mathbf{R}^*$, $\Delta = (2m - 1)^2 - 4m(m - 1) = 1 \Rightarrow x_1 = \frac{-(2m - 1) - 1}{2m} = -1 \in \mathbf{Z}$, pentru oricare $m \in \mathbf{R}^*$;

dacă $m = 0 \Rightarrow -x - 1 = 0 \Rightarrow x = -1 \in \mathbf{Z}$.

15. b) $\triangle DAM \cong \triangle ABN$. Deci $\widehat{ADM} \cong \widehat{BAN}$, $\widehat{DMA} \cong \widehat{ANB}$. Dar $m(\widehat{ADM}) + m(\widehat{DMA}) = 90^\circ \Rightarrow$

$m(\widehat{BAN}) + m(\widehat{DMA}) = 90^\circ \Rightarrow m(\widehat{APM}) = 90^\circ \Rightarrow AN \perp DM$ ($DM \cap AN = \{P\}$).

c) $A_{\text{totală}} = 96 \text{ cm}^2$.

d) $A_{\triangle A'MD} = \frac{DM \cdot A'P}{2} = \frac{2\sqrt{5} \cdot \frac{4\sqrt{30}}{5}}{2} = 4\sqrt{6} \text{ cm}^2$. $9 < 4\sqrt{6} < 10 \Leftrightarrow 81 < 96 < 100$ adevărat.